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FOUNDATIONS OF A THEORY OF ECONOMIC WARFARE AND ARMS CONTROL

Murray Wolfson and John P. Farrell*

ABSTRACT. War is carried out even in "peacetime" by exerting economic pressure in the form of forced deterring expenditure as well as by military threat. Peace can achieved only by considering both economic and military arms control. In this paper, disarmament and its verification by fiscal control are studied as a means of negotiating a reduction and elimination of both dimensions of war. This is first done under simplifying assumptions, but then is followed by an analysis of the complete theory of economic and military interactions.

A. TOWARD A NEGATIVE ARMS RACE

1. INTRODUCTION

In previous papers (Wolfson, 1985; Wolfson and Farrell, 1987), we have explored the implications of the interdependence of military and economic war. The perceived need of nations to deter one another from physical attack implies a matching of military power, and through it the diversion of resources from civilian to military purposes. Large military allocations by one country affect the resource allocations of its opponent and, given limited production possibilities, impose costs on it. This is especially so when the factor endowments of the antagonists differ. Then deterrence may require a nation to violate the law of comparative advantage to prepare to fight on the enemy's terms and thus to make intensive use of resources which it finds relatively scarce. The relation is symmetric since both antagonists may simultaneously threaten the other with increased costs in this fashion. We have argued that much of the recent history of the relations between the U.S. and U.S.S.R. can be interpreted in his this light.

Since economic considerations are central to the present arms race, new importance must be attached to the long-standing United Nations proposals to control the arms race by a military expenditure limitation (MEL) treaty rather than the present weapon-by-weapon bargaining. MEL proponents have held such a comprehensive approach to be more effective, since weapon-by-weapon agreements tend merely to redirect

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*Professor, Department of Economics, California State University–Fullerton; and Professor, Department of Economics, Oregon State University.

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the arms race to other, necessarily more expensive permissible forms (Becker, 1977; Holzman, 1975).

In Part A of this paper, we attempt to put this issue in theoretical perspective. We model a highly simplified version of MEL abstracting from many of the problems of implementation that have exercised the experts.[1] We show that MEL economizes on information that must be obtained by direct surveillance, and in this sense is no less practical that the direct-agreements approach. We show, further, that the kind of agreement we propose generates economic forces which encourage reductions in arms production, followed by further agreements and reductions, and thus makes possible the initiation of a negative arms race.

Despite its attractive optimization features, the exercise in Part A shows that MEL, like any agreement or disagreement, cannot be understood apart from the overall choices and purposes of the nations involved. In Part B, we propose the outline of a dynamic (two-period) interactive optimization model that can be applied to the U.S.-U.S.S.R. relation. In this fashion, we attempt to define the conditions in which a negative arms race might or might not take place, and thus suggest some of the reasons for the persistence of the arms race over four decades of postwar history. That is to say, we study possibilities for negotiation toward arms control on one hand and the reasons for economic and military war on the other.

2. THE MILITARY-POWER FUNCTION

Our previous efforts to integrate military activity into their social context stemmed from our study of the economic limits of the missile arms race portrayed in the models of Intriligator and Brito (1983). It became clear that our inquiry required a macroscopic concept of military power more general than a unidimensional missile count which represents only a part of military expenditure. We posited a military-power function (MFP), taken to be a scalar mapping from all the military instruments of each country to some measure of force which it could pit against a standardized opponent.[2] Military power for each country was a sort of generalized missile whose effect depended solely on the weapons of the *given* country, the standard opponent being held constant.

Whatever the defects of this procedure, it permitted us to consider the inverse mapping from that power to the military instruments which were classified grossly as "high technology" (H) and "low technology" (S). In turn, these systems of weapons and personnel could be traced back to the resources—the factors of production—allocated to their production and maintenance. Insofar as the resources used are large compared to a nation's total stock, they reflect its factor endowments both in size and composition. Mutual deterrence could then be interpreted much more generally than missile parity: deterrence is the achievement of the same "isoquant" of military power by both parties, even though the instruments and therefore the resources required to do so might differ between them reflecting their comparative advantage.[3]

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[1] A careful discussion of these problems can be found in Becker (1977, Chapter 3) and in various U.N. reports [e.g., U.N., (1982)].

[2] MPF corresponds to Becker's (1977, p. 17) "military security function" or his "force potential." Becker sees that this function will have to be evaluated in utility terms but despairs of that prospect, in our opinion, too soon.

Yet military power remains a subtle and elusive concept. Military power may be understood in the sense used above as if it were a technical military relationship with meaning as an ordinal function: for a given common state of the military arts and no meaningful military secrets, for the given combination of H and S of each country, country A has more military power than B if it deters B from attacking A; A has less than B if B deters A; A and B have equal military power when they mutually deter one another. The isoquants of military power, which do not intersect given the conditions specified, represent combinations of weapons that have the same capacity to deter attack.

Even at equilibrium, it is clear that whether a nation is deterred or not depends on some subjective measure of the good or harm that might be done by attacking or by allocating resources to warfare. It is apparent, then, that to speak meaningfully about military power as a social phenomenon we need to evaluate the effectiveness of the instruments in terms of their impact on people; that is to say, we need to relate these "generalized missiles" to their influence in intensive magnitudes on the level of public utility. Our concept of military power requires a broader conception of utility in which the security of life and property, consumption now and in the future are arguments of the utility of society or its decision makers.

The difficulty is that each nation does not make its decision independently of the other. It may be that under conditions of hate, the elements of utility suggested might also include satisfaction gained from the *disutility* of another. Even without active "hate," the fact is that weapons can be used for attack as well as defense. Hence, the military and economic power of one country and the utility level it achieves also constitutes a threat to the security of its opponent and thus enters as a *negative externality* into their interactive optimization process.

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[3] As suggested above, when the military-power function is not well-defined, each nation may have to fight in terms of the MPF of its opponent, and hence the security of deterrence is undermined and with it the possibility of employing the law of comparative advantage to achieve static equilibrium. See Wolfson and Farrell (1987).

Military power, then, turns out to be a complex aspect of an interactive two-party dynamic general "equilibrium" process that has subjective social as well as technological military aspects. We shall, as promised, return to these issues after offering a simple—we think improved—MEL model of arms control.

3. AN EXERCISE IN ARMS CONTROL

We first carry out a naive exercise in MEL arms control, assuming present approximate military parity, and a desire on both sides to reduce the threat level. We consider a system of expenditure control (and pricing of arms) freeze and then successively reduce military expenditures and threats.

In this first exercise, we retain the MPF in which each country's military power depends on its own level of H and S. For the moment, we can limit our discourse to the ordinal properties of the MPF since, in this section, we focus on the comparative statics of deterrence situations in which military power is equal. Let us take it that MPF is twice differentiable and at least quasi-concave.

Let us make simplifying assumptions about information: the two nations have no military secrets from each other[4]; they know each other's H and S as well as their overall economic condition; and they share a common knowledge of the state of the military arts. However, military knowledge is inexact, so that *nobody* can foretell the outcome of war with certainty. Although something of a misnomer, Von Clausewitz (1832–1837) called this uncertainty "friction."

The greater the friction, the more difficult it is to define MPF, and consequently achieve stable deterrence. In our earlier paper, we discussed the destabilizing character of this information gap, including the possibility that it might not be possible for the parties to imagine a well defined MPF for widely differing combinations of instruments. Indeed the function might not exist. We also showed how countries attempting to estimate military power by comparisons of expenditure tend to bias their calculations upward as a result of the working of the classic index-number problem in which each nation estimates the expenditures of the opponent in terms of what it would have to spend itself—in its *own* price system—to reproduce the opposing military allocations of H and S.

We take it that friction is greater when there is a greater difference in the way in which countries prepare for war—their ratios of H to S. In the limiting case of identical ways of fighting—remembering that at this level of abstraction—we do not yet consider subjective evaluations of the costs or benefits of war. *Victory* simply goes to the greater numbers, and deterrence requires equal numbers just as in the Intriligator-Brito missile count.[5] Failing this, the informational problems which a MEL treaty must address are (1) the verification and control of arms acquisitions and (2) the reduction in expenditure in the face of Von Clausewitz's friction and the need for continued deterrence.

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[4] More realistically perhaps, they can acquire any desired information by a sufficiently high allocation of H and S to intelligence operations.

Suppose the factor endowments of the countries differ significantly, the U.S. being relatively capital-abundant and the U.S.S.R. relatively labor-abundant. Suppose military expenditures are large and, therefore, reflect the different endowments. Then, the combinations of H and S that each can deploy for given resources allocated to the military are defined by a *military production possibility frontier* (MPPF). MPPF is downward-sloping and concave toward the origin for the usual reason of diminishing returns. On the further assumption that neither country dominates[6] the other, so that it cannot produce a greater number of *all* weapons, the MPPFs must intersect. Mutual deterrence occurs when an isoquant of the MPF is tangent to the MPPF of each country at A_X and A_Y as shown in Figure 1. The slope of the rays from the origin to these two points indicates the relative mixtures of military instruments each employs. The ray OM is the common mixture of instruments consistent with both MPPFs.

While the idea of military-expenditure limitation (MEL) is hardly new—it has been proposed and debated intensively in the U.N. since 1973—arms-control agreements to date have been specific force limitations.[7] Yet MEL has a number of advantages. The information required for verification, while formidable, is clearly less than what would be required for verification of a broad, direct force by force agreement. Second, MEL agreements are inherently comprehensive. In practice, partial weapon-by-weapon negotiations and treaties often serve to define the domains of new arms races in even more costly permitted systems.[8] MEL agreements afford countries "freedom to mix" military

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[5] "Victory" in some of the game literature means the ability of one of the parties to "enact any feasible alternative" (Laing and Slotznick, 1987, p. 65). This will do for our purposes in Part I, but clearly economic and military wars are more properly seen as continuous games in which both parties experience and impose costs and benefits measured intensively along some utility index of their own. This consideration is deferred until Part II of this paper.

[6] Country X "dominates" country Y if the military power of X is greater than that of Y for any military power function and for any combination of S and H; the MPPF of Y lies wholly within that of X (Wolfson and Farrell, 1987).

[7] For a documentary history of military expenditure limitation (MEL) proposals, see Becker (1977, pp. 91ff).

[8] For instance, in commenting on the abortive Iceland summit, Secretary of Defense Weinberger remarked that if ICBM's were abolished, the U.S. would have to build up a much more expensive conventional deterrent. If Americans felt that they were not willing to live with push-button deterrence, then they had to be willing to pay the price. Neither the danger of war nor the reduction of cost seemed to be reduced by the partial progress

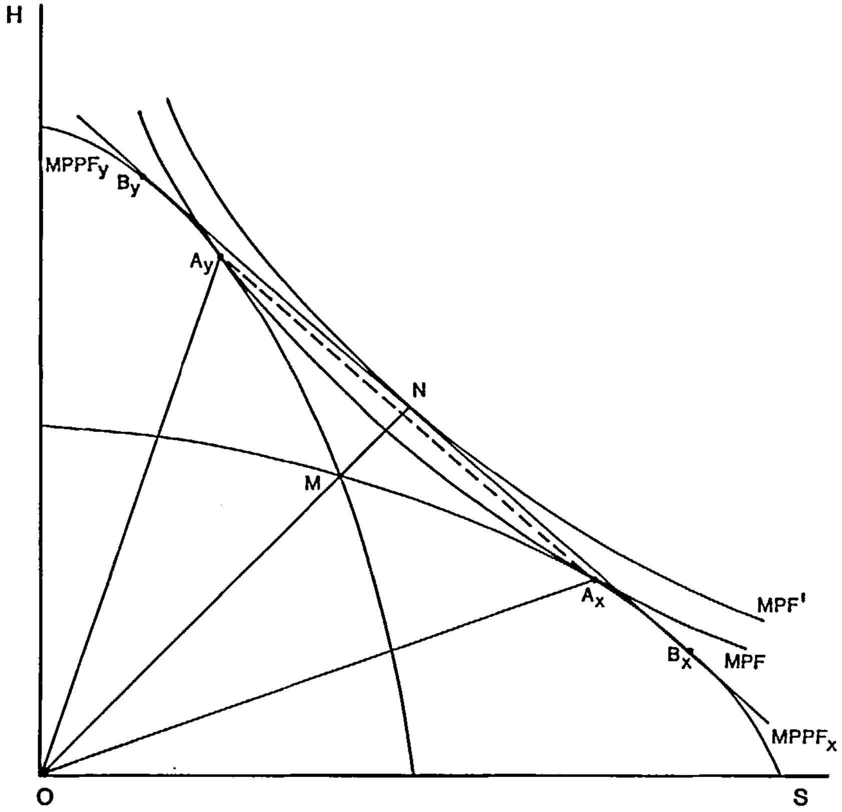

FIGURE 1: Military Power Function (MPF) and Military Power Production Possibility Frontier (MPPF).

instruments, so long as the macroeconomic constraint on total expenditures is met. Third, aggregate control and freedom to mix permits efficient choice in the production of military power in accord with each country's technology, factor endowment and relative prices.[9] Finally, if we are right that much of the danger comes from economic warfare, the comprehensive MEL agreement would also prevent this form of hostilities from escalating out of control.

Of course, the U.S.S.R. would have to be much more forthcoming than it has been in providing detailed information on both its civilian and military expenditures. Certainly the Soviets realize that an arms

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that was thought to have been made (McNeil-Lehrer News Hour, Public Broadcasting System, October 28, 1986).

[9] It might be necessary as a practical matter to make some direct agreements to limit the most dangerous weapons even though in principle this "linkage" is suboptimal. (Becker, 1977, pp. 57–59.)

agreement of any kind requires that they provide information which they would rather not divulge (Cooper, 1989). Recent work by the U.N. and others has made clear what is needed for effective verification of MEL agreements.[10] Both sides would have to providing detailed budgetary and financial information, including time series to facilitate detection of hidden military expenditures in loans, nonmilitary expenditures, residuals, etc., as well as access to foreign samples, surveys and audits (Holzman, 1975). Radical by historical standards, this kind of access would test Gorbachev's policies of "openness." A special valuations problem arises with respect to the U.S.S.R. because of the tax and subsidy schemes peculiar to centrally planned economies. Fortunately, the "adjusted factor cost" method pioneered by Abram Bergson to convert to Soviet national income statistics to a facsimile of their U.S. counterparts could be used in evaluating Soviet military expenditures (Becker, 1977, pp. 23–24).[11]

The central question is the *price* system by which the budgetary limitation in money will be translated into weapons systems and military power. In order to gain insight into this problem, let us first consider the case of two "small" countries facing the same problem under circumstances in which there is an international market in armaments and, more awkwardly, personnel. The situation is depicted in Figure 1 which displays the standard international trade theory result.

Under autarky, mutually deterring countries could produce and deploy their military instruments at their mutual points of tangency of MPPF with the isoquant of MPF at A_X and A_Y. Shadow prices would be given by these same points. Under conditions of free trade in which they could trade military instruments at world prices (which we presume to be intermediate between the shadow prices), they would arrive at the more highly specialized production points B_X and B_Y respectively. They would then trade H and S until they reached a market clearing point, such as N, at which they would both be at a higher level of military power but would continue to deter each other.

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[10] U.N. work on uniform reporting and verification of military expenditures is well-advanced, and several countries—although not the U.S.S.R.—have been reporting such expenditures according to a standardized "instrument" (United Nations, 1982). Alternate early variants of these accounting models for military expenditures are discussed in Becker (1977, pp. 27–37). After noting in their introduction to the 1985 Report of the Group of Experts on the Reduction of Military Budgets commissioned by the Secretary General of the U.N. that the "basic problem would be to agree on common definitions and rules for valuation, accounting and reporting of the military expenditures of the negotiating parties," Cars and Fontanel (1987) conclude from their studies of market economies that good international comparisons are possible with a relatively limited amount of data" and that these could form the basis for MEL agreements.

[11] This valuation problem can be avoided by the treaty if prices are negotiated as we propose.

There are important gains in this situation despite the higher level of military power because the *information content* of the new situation is increased: Since both countries achieve their military power with the same instrument mix, the slope of the ray ON, Von Clausewitz's friction is reduced. Since the prices are identical (military budgets are comparable no matter what currency is used), the index-number problem vanishes. If the functions are homogeneous, military-expenditure reduction can be accomplished by an agreement to reduce all weapons proportionately.[12] This process can be generalized to multilateral disarmament programs.

Now we must consider the case in which there is little arms trade between countries of differing social systems such as the U.S. and U.S.S.R. We propose a two-stage arms-reduction process illustrated in Figures 1 and 2.

The first stage would require a budget freeze and the negotiation of international accounting prices in terms of a newly created unit of account, somewhat akin to the special drawing rights (SDR). We may call it a PAX. Each country would initially be permitted sufficient budget in PAX to maintain its present military establishment. In terms of Figure 1, the PAX prices (the unique set consistent with present military expenditures of both sides) is given by the slope of a line connecting A_X and A_Y. This line is the *budget-freeze line.* If the functions are all homogeneous, the budget-freeze and international terms-of-trade lines would be parallel and (in the long run) sloped equal to the relative prices given by the purchasing power parity (PPP) method. But that is too much to expect; we would anticipate long and arduous negotiations are to the relative accounting prices, although international arms markets and PPP methods might serve as a starting point. Long accustomed to negotiated prices based loosely on world prices in the context of international trade, the Soviets should find nothing strange in this procedure.

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[12] The U.N. procedure (United Nations, 1982) evaluates military expenditures in terms of factors of production used, but must face the discrepancy between the value of the factors in each country due to their different endowments and allocations. It applies the classical purchasing-power parity doctrine of exchange rates (PPP); in full equilibrium the exchange rates of countries reflect their market determined factor pricing and productivity in producing their chosen commodities. This procedure may be applied to market economics, for which, in any case, the problem is made easier by the tendency for markets in arms to lead to convergence of military instruments. The method becomes less suitable for the U.S. and U.S.S.R. who choose different combinations of both civilian and military goods and are not nearly small market economies buying their arms in fully equilibrated international markets. Nevertheless, we believe that the form of MEL which we will propose can function because pricing is to be negotiated between the parties specifically to maintian deterrence.

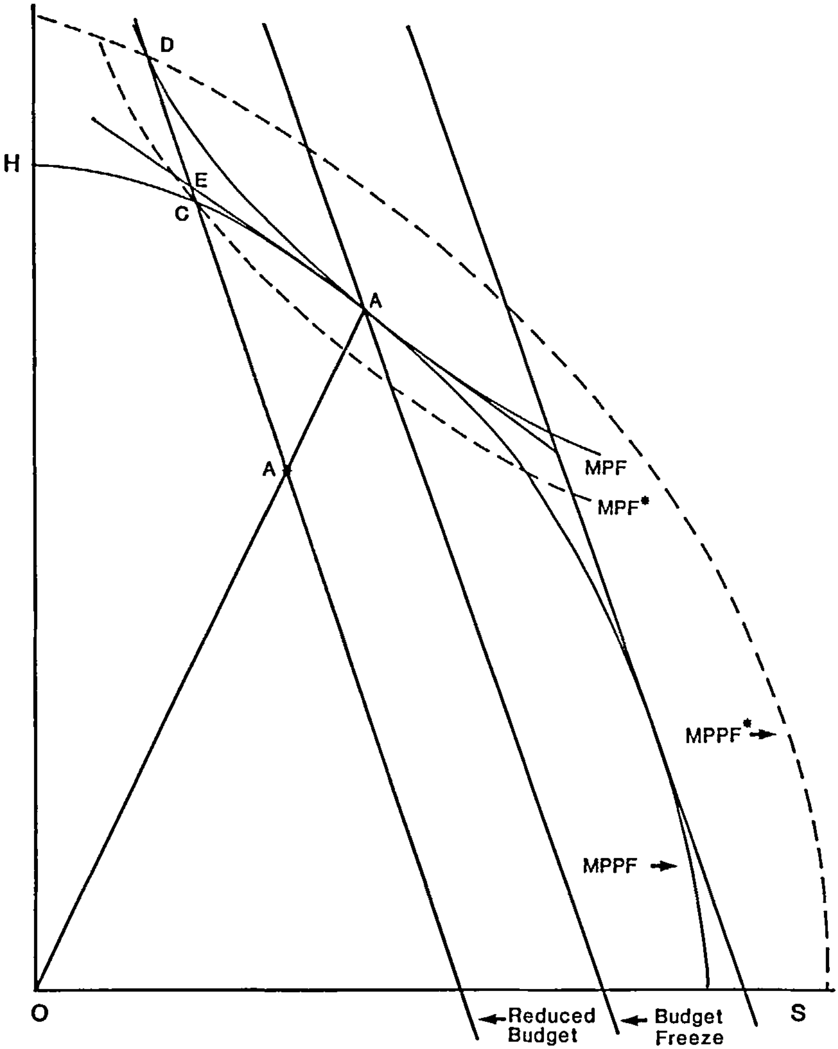

FIGURE 2: Effect of Military Expenditure Reduction.

The most important points are

(1) The PAX prices would be intermedite between the shadow prices of each country.[13] In their bargaining stances, both sides would attempt to maximize the PAX price of the *H* and *S* in which they tend to specialize in order to enlarge their permissible budget. This proce-

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[13] We are assuming that each country is efficient, maximizing MPF subject to MPPF, thus arriving at the correct shadow prices given by the mutual tangency between MPPF and the highest isoquant of MPF.

dure will still permit them to obtain the same amount of their advantaged instrument and have the opportunity to enlarged expenditure on the other.

(2) At the first stage the mutually deterring military establishment at A_X and A_Y should remain intact. By hypothesis, this is the maximum military power feasible for each country with the given allocation of real resources in MPPF that also satisfies the budget freeze. We are careful to notice that the optimization is carried out in response to the different national *shadow* prices not the PAX prices for which it is sub-optimal. The budget-freeze line is not a tangent but a chord of MPPF.

The second stage is a reduction in budgets at the agreed upon PAX prices as illustrated for country X in Figure 2. The functions are as in Figure 1 but omit subscripts. Presumably this is the U.S.S.R. with a comparative advantage in *S* relative to *H*. Resource reallocations by one side will be just matched by the other, so that the deterrence condition always holds. The reduced-budget line, parallel to the budget-freeze line, makes A infeasible but leaves open many alternatives.[14] Let us consider four of them.

(1) We study the worst case first. Country X might choose point D, the tangency point of the reduced-budget line and the MPF isoquant. D yields the same military power as A but represents a higher cost at real internal shadow prices, requiring that more resources be directed to the military sector by moving the MPPF outward to MPPF*, just sufficient to reach D. Our system of pricing has forced X to achieve D inefficiently in real terms, thereby effectively taxing its military power. While deterrence is maintained, it is purchased at a higher real cost in foregone consumption. Since X was already optimizing MPF subject to its military and civilian resource allocation at A, it is not reasonable to assume that it would choose D to get the same result at a higher real cost. Economic incentives will push in the direction of smaller military

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[14] Becker (1977) discusses several cases where he feels that reallocation of military expenditures leads to greater insecurity, as for example an allocation from current force maintenance to research and development, which might set off a "qualitative" arms race, or from intelligence gathering to procurement, which might heightened the possibility of a preventive strike; or when the tighter military-budget forces out organizational slack, thus improving efficiency but perhaps differentially and thus in destablizing way. Yet these possibilities are always present, and there is no reason to believe them to be more dangerous under MEL. After all, except for propaedeutic purposes in Part I, we are not considering a disarmament program departing from a static deterrence equilibrium which we dare not disturb, but from an unstable disequilibrium arms race that takes on the character of economic warfare which may develop into military war unless checked. If anything, MEL should have a salutary influence on the time shape of the arms race, since a country falling behind in the economic war might well accelerate its military acquisitions and launch a preventive war while it still has the chance.

power. Nevertheless, it must be conceded that X might choose D, and country Y might respond in kind. Then the economic war would be tightened by one—but only one—notch within the bounds of the MEL treaty.

(2) Country X might choose C, the intersection of MPPF with the reduced-budget line. Once again the treaty taxes MFP, because the country would be committing the same real resources in MPPF but achieving only a lower level of military power, MPF*. This is so because C is inefficient.

(3) Country X might choose E, the intersection of the internal budget line through A after the freeze with the international reduced-budget line. X obeys the treaty using the negotiated accounting prices but spends the same amount of money in terms of its internal system of shadow prices. This alternative is intermediate between D and C both in terms of MPF and internal real and money costs. (The enlarged MPPF and internal budget lines at E are not drawn in Figure 2.)

(4) Although it is not a point of maximization, country X might attempt to avoid the incongruities of the effective tax on military power by choosing a point like A*, interior to the present MPPF and, by assumption, just consistent with efficiency in military-power production and the reduced-budget line.

Of course, the fact there are alternative actions that might be taken by each country within the confines of the proposed MEL program shows that this is not a determinate system. There is still room within it for continued action and reaction between the parties. Nevertheless, the following general points can be made:

(1) A budget reduction on the basis of compromise accounting prices unambiguously raises the cost of any given level of military power and therefore promotes a reallocation of resources towards civilian uses.

(2) The proposed treaty, for all points illustrated in Figure 2 (other than A*), leads to a military instrument mix on the part of each country which more closely resembles that of its opponent. Thus while it does not achieve an international trade equilibrium at N in Figure 1, it does increase the informational content about the MPF, which should inspire greater confidence in the agreement and lay the groundwork for a second round of reductions.

(3) Assuming choice close to C in Figure 2, the process of convergence might continue with further budget cuts until—perhaps fancifully—a point is reached like M in Figure 1 where both frontiers intersect and the countries are producing an identical military establishment and level of military power. Further reduction might then be achieved by direct proportionate reductions in all military instruments as in the case of traded military instruments in the "small country" case discussed above.

A negative arms race based on economic self-interest and increased verifiability and evidence of good faith seems too good to be true. But why? At first one might be daunted by the sheer statistical and negotiating burden of defining and pricing all the military instruments. Yet that is less difficult than an arms control agreement dependent upon evaluating these same instruments directly as they might be used in generating the MPF under all hypothetical circumstances. Moreover, the effectiveness of the agreement would exceed the partial arms-control treaties which, as we have said, seem neither to control military power nor its costs very effectively.

The problem goes deeper. The interdependent nature of military power prevents us from rigorously defining the MPF as a function of a nation's *own* military instruments against a static, standardized opponent. Military power must reflect the influence of feedback mechanisms between the countries as evaluated in their respective utility functions. We have to consider that these utility functions may contain more then consumption and self-defense. If utility functions are interdependent, it is more likely that the tie the binds is hate rather than love.

So we have to make an attempt at a full interactive model. We can hope only to sketch the outlines in this paper, linearizing where convenient. We hope to come to a deeper understanding of the conditions under which MEL proposals are viable. They are not easy conditions and MEL is not a panacea which when once thought of will reconcile hatred and totally eliminate problems of information and instability. But MEL is a good way for parties anxious to come to agreement to do so. If MEL founders on these rocks, it is likely that other methods will as well. Happily the converse of this proposition need not hold. If the basic obstacles to a peaceful agreement are removed, weapon-by-weapon arrangements might be negotiated with implications for military expenditures that limit the economic war. MEL would be accomplished as the dual, so to speak, of physical weapon limitation. In any case, comprehensive consideration of economic and military factors seems to be a prerequisite for an effective reduction in international tension.

B. A GENERAL MODEL OF ECONOMIC AND MILITARY WAR

1. INTRODUCTION

The outline of a two-period dynamic optimization conflict model is shown in the flow chart (See Figure 3). Each nation X and Y is endowed in year *t* with a resource stock of capital (K), population (P), and natural resources (N) which it allocates to the civilian and military sectors. Its labor supply is equal to the population times the labor-force participation rate (p).

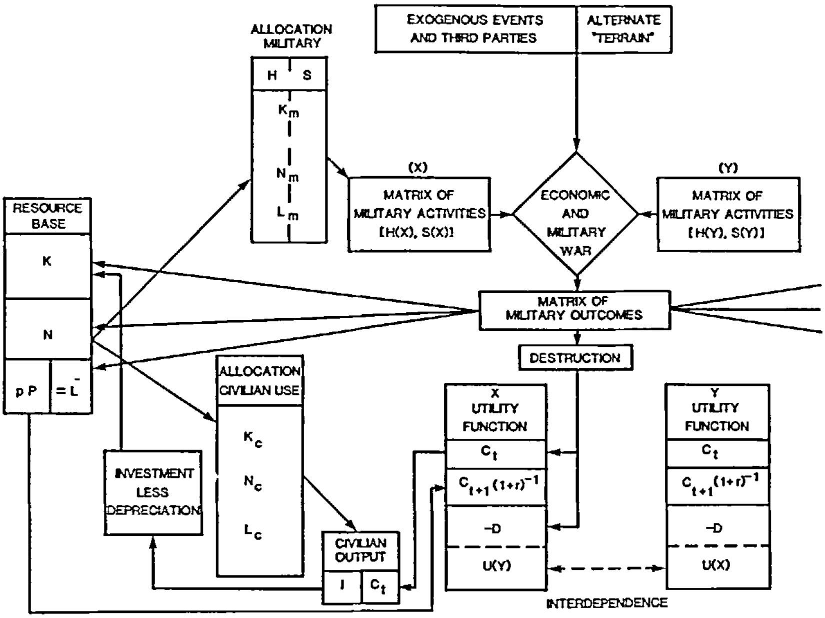

FIGURE 3: Interacting Military and Economic Activities of Opposing Countries.

Corresponding to each possible allocation of available resources, it is feasible for the civilian and military sectors are able to produce a corresponding columnar list (vector) of feasible products and activities. Civilian outputs are consumption and (gross) investment goods and service activities. The table collecting all the columns of feasible civilian output vectors is the matrix of civilian production. Resources allocated to the military sector generate a matrix made up of all the feasible military-activities vectors.

Each of these vectors will be a list of different amounts of various activities. We will assume that there are a limited number of kinds of civilian and military products so the vectors are finite dimensional. But since each different combination of amounts of output constitutes a different vector, there may be an infinite number of these finite dimensional vectors as the amounts of outputs vary continuously. For expository purposes, we will consider the military-activity vector to be two dimensional and consist of H and S as defined previously. Of these, efficiency requires that any vector is excluded from consideration if there are other vectors such that the second vector includes all of the amounts of the activities of the first and more of at least one activity.

The set of all possible military-activity vectors of each country is fed into the diamond labelled economic and military war. They interact in this arena along with the set of all possible terrains and exogenous conditions including the activities of third parties. Corresponding to each combination there results a set of vectors of *military outcomes.* As long as the list of activities is finite, and the conflict is a well-defined function of these arguments, the military engineers can be called upon to predict the set of military outcomes—at least in principle. These outcomes determine the number of casualties, the reduction in consumption and the destructon of human, natural, and capital resources.

That is as far as we can regard war as a technical military matter. Each of these alternate military outcomes must be evaluated in terms of the utility function of each country. We no longer separate the purpose of war from other national purposes. It may not be the case that the conduct of war demands the single-minded goal of destruction of the military power of the enemy [as Von Clausewitz (1832–1837) also suggested in contradiction to his earlier dictum about the political goals of war]. Rather, each nation makes utility-maximizing decisions regarding economic and security goals in the dynamic conflict situation they find themselves.

Whether these decisions will be consistent with one another, whether they will spell peace or war, negotiation or impasse, what their dynamic interactions will be—all depend on the data and the functional specifications. We examine two cases. In the first, the decision makers are rational in the sense that they are concerned only with the national consumption in the present and future years and the safety of their own population. In the second, there is an interaction between the utility functions in the form of active hate. Each nation is better off, it feels, if the other is worse off. Let us specify the simplest "rational" case first, and analyze it carefully, leaving the case of interdependent "hate" for detailed study on another occasion.

2. THE SOCIAL WELFARE FUNCTION

The social welfare function (SWF) which we shall discuss is the utility function of the national decision makers of countries X and Y reflecting their evaluation in time period t of the condition of the population of their own country, say country X, $U_{X,t}$. The arguments of this function are: aggregate consumption in year t, $C_{X,t}$; consumption next year discounted to the present at the social rate of discount, r, $(1 + r)^{-1}C_{X,t+1}$; and a measure of public safety, $-D_{X,t}$, where D represents the number of casualties in year t. Then the utility of X is

$$U_{X,t} = U[C_{X,t}, (1 + r)^{-1}C_{X,t+1}, -D_{X,t}] \tag{1}$$

Henceforth, where appropriate and advantageous, the country index will be dropped. Where no time index is indicated, period t will be understood.

U is understood to be concave and twice differentiable in the *open* set over which it is defined. All the first partial derivatives are positive. For most "normal" economics models, the utility function is taken as continuous over some *closed* and *bounded* set of arguments. Whatever the validity of this procedure, it is not appropriate to many political models, much less one which includes war as one of its outcomes. These models are subject to catastrophes or discontinuities especially as represented in the SWF of leaders as they reflect national interests as filtered through their personal ambitions and perceptions. The definition of the domains of U to the set of arguments between but not including catastrophies means that U must be defined over an *open* set. This is equivalent to saying that equilibrium optimization of utility may not exist under circumstances which can bring about sharp irreversible changes in regime, social structure, or political power ranging from the fall of a parliamentary government to military defeat.

For those portions of this paper dealing with static equilibria of mutual deterrence, we may only need to consider U as an ordinal relationship. However, we will be forced to consider cardinal measures of utility when dealing with disequilibrium processes and some versions of interdependent utility.[15]

3. ALLOCATION

(i) The initial resource endowments at the beginning of the process constitute a set of predetermined variables given at t. They are the capital stock, K_t; natural resources, N_t; and the labor force, L_t. The labor force is equal to the labor-force participation rate times the population minus casualties for the preceding year, $t - 1$.

$$L_t = p(P_{t-1} - D_{t-1}) \tag{2}$$

If military war should start in t, $P_t = P_{t-1}$ and $D_{t-1} = 0$, so that $L_t = pP_t$.

(ii) Putting problems of joint products aside to postpone lengthy exercises in the chain rule, resources are allocated to civilian (E) and military (F) use in period T, such that $T = (t, t + 1)$. Then we have the identities

$$K_T = K_{ET} + K_{FT} \qquad N_T = N_{ET} + N_{FT} \qquad L_T = L_{ET} + L_{FT} \tag{3}$$

4. PRODUCTION

Civilian products in year T are consumption, C_T, and gross capital formation, I_T. Their production functions are

$$C = c(K_C, N_C, L_c) \qquad I = k(K_I, N_I, L_I) \tag{4}$$

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[15] There is no completely satisfactory way of achieving this, but we can introduce some realism into the model by assigning utility proportional to the perceived probability of catastrophe.

where K_C is the allocation of $K_{E,T}$ to the production of consumption goods and K_I is the allocation of $K_{E,T}$ to the production of capital goods and so on.

The military activities vector of goods and services are aggregated into high technology, H_T, and low technology, S_T, produced according to the production functions

$$H = h(K_H, N_H, L_H) \qquad S = s(K_S, N_S, L_S) \tag{5}$$

where K_H is the allocation of $K_{F,T}$ to the production of high-technology military goods, and K_S is the allocation of $K_{F,T}$ to the production of low-technology military goods, and so on.

5. DEPLETION

Capital depreciates at d percent of the capital stock, and natural resources are depleted at q percent of the stock of natural resources. Of course, these rates are properly related to the GNP rather than to the stock of the resources, but the assumption that the ratios of output to capital and natural resources are roughly constant permits this simplifying formulation.

6. MILITARY OUTCOMES

In general, the outcomes of economic and military war may be described as a two-party nonzero-sum game which could be represented in matrix terms for pure strategies corresponding to military activities (H, S) played by each actor in the conflict. Thus there is a matrix of outcomes **O** which may be written as

	H_Y	S_Y
H_X	$\mathrm{O}_{HH\mathrm{X}}, \mathrm{O}_{HH\mathrm{Y}}$	$\mathrm{O}_{HS\mathrm{X}}, \mathrm{O}_{HS\mathrm{Y}}$
S_X	$\mathrm{O}_{SH\mathrm{X}}, \mathrm{O}_{SH\mathrm{Y}}$	$\mathrm{O}_{SS\mathrm{X}}, \mathrm{O}_{SS\mathrm{Y}}$

In the present context, we are dealing with a continuum of alternative mixed strategies combining the military activities in various proportions. Corresponding to each of the combinations there will be military outcomes for each nation such as the human casualties, the destruction of capital stock and natural resources and the disruption of consumption. These outcomes may be zero either because the values of H and S may be zero, or because positive levels of military instruments directed against opponents may not necessarily result in active war. It will prove useful to designate each of these outcomes as vectors of the outcome for nations X and Y. In general for an outcome ($\mathbf{O}_\mathrm{X}$, $\mathbf{O}_\mathrm{Y}$) we have the expression

$$(\mathbf{O}_\mathrm{X}, \mathbf{O}_\mathrm{Y}) = \mathbf{F}(H_\mathrm{X}, S_\mathrm{X}, H_\mathrm{Y}, S_\mathrm{Y}) \tag{6}$$

We can substitute the particular outcomes for **O** and the names of the destructive military outcomes for **F** according to the following table:

	Outcome	Function
(7)	Capital destruction	$Z = z(H_X, S_X, H_Y, S_Y)$
(8)	Casualties	$D = g(H_X, S_X, H_Y, S_Y)$
(9)	Consumption disruption	$W = w(H_X, S_X, H_Y, S_Y)$
(10)	Natural-resource destruction	$R = r(H_X, S_X, H_Y, S_Y)$

7. CONSTRAINT IDENTITIES

The use (demand) of each of the factors of production in producing civilian and military goods and activities for country i ($i = X, Y$) in year T ($T = t, t + 1$) is given by

$$K_{iT} = K_{iCT} + K_{iIT} + K_{iHT} + K_{iST} \tag{11}$$

$$N_{iT} = N_{iCT} + N_{iIT} + N_{iHT} + N_{iST} \tag{12}$$

$$L_{iT} = L_{iCT} + L_{iIT} + L_{iHT} + L_{iST} \tag{13}$$

The corresponding stocks in period T are given by

$$K_{iT} = K_{i,T-1} - dK_{iT} + I_{iT} - Z_{iT} \tag{14}$$

$$N_{iT} = N_{i,T-1} - qN_{iT} - R_{iT} \tag{15}$$

$$L_{iT} = p(P_{T-1}) - D_{iT} + Q(C_T) \tag{16}$$

$Q(C_T)$ reflects the influence of real income (consumption) in motivating a change in the labor-force participation rate and productivity levels.

8. EVALUATION

Each country is faced in year t with maximizing its utility over the current year and year $t + 1$ ("today" and "tomorrow," respectively). Restricting ourselves to interior solutions, we search for the constrained maximum chosen by each party under circumstances in which the resource constraints depend on the actions of its antagonist which may either destroy (military war) or divert them (economic war). Consequently, in forming the Lagrangian we must map both the objective functions of each nation and the constraint systems into the underlying allocations of resources by both of them into a vector **F**. Since there are two countries (X, Y) each with three resources (K, L, N) used for four purposes (C, I, H, S), there will be 24 such allocations.

In order to make this chain of relationships at all transparent, we propose for expository purposes to linearize them locally so that both the utility functions and the constraint system will consist of separable terms corresponding to each of the 24 allocational elements. Vector and matrix quantities will be written in boldface to distinguish them from symbols used elsewhere in this paper. To preserve concavity in the utility function, it may be supposed that we have transformed its arguments into their logarithms.

(i) *Utility Function.* The utility function of each country ($i = \mathrm{X, Y}$) is written in separble form as the inner product of the vector of marginal utilities of its arguments (**MU**) times the vector of the three components (C_t, C_{t+1}, $-D_t$) which we call $\mathbf{A}_i$

$$(17)\quad U_i = [\mathbf{MU}_{C(t)}, \mathbf{MU}_{C(t+1)}, \mathbf{MU}_{-D}]' \begin{bmatrix} C_{t,i} \\ (1+r)^{-1}C_{t+1,i} \\ [-D_{t,i}] \end{bmatrix} = \mathbf{MU}'\mathbf{A}_i$$

The vector **A** contains the elements for both X and Y and hence has six entries. $\mathbf{A} = (\mathbf{A}_\mathrm{X}, \mathbf{A}_\mathrm{Y})'$.

The values in **A** are given by a composition of matrices of coefficients involving the production of the elements of **A** minus their partial destruction all related to the vector of 24 allocations of the two countries. We define each of these matrices in turn

(ii) *Matrix* **B***: Production Coefficient* (see Figure 4). The technical coefficients of production of the elements of **A** for both countries (6 in all) is given by the 6-by-24 matrix **B**. The 6 rows show the 3 elements of the utility function for each country and the 24 columns represent the 12 factor allocations considered for each country. We would expect that this matrix would be largely composed of zeros since only the productive activities enter here marked by "#." Unmarked terms are zero.

Country X Resource Allocations

	1	2	3	4	5	6	7	8	9	10	11	12
	K_C	K_I	K_H	K_S	L_C	L_I	L_H	L_S	N_C	N_I	N_H	N_S
$C_{t,\mathrm{X}}$	#				#				#			
$(1+r)^{-1}C_{t+1,\mathrm{X}}$	#	#			#	#			#	#		
$-D_{t,\mathrm{X}}$												
$C_{t,\mathrm{Y}}$												
$(1+r)^{-1}C_{t+1,\mathrm{Y}}$												
$-D_{t,\mathrm{Y}}$												

Country Y Resource Allocations

	13	14	15	16	17	18	19	20	21	22	23	24
	K_C	K_I	K_H	K_S	L_C	L_I	L_H	L_S	N_C	N_I	N_H	N_S
$C_{t,\mathrm{X}}$												
$(1+r)^{-1}C_{t+1,\mathrm{X}}$												
$-D_{t,\mathrm{X}}$												
$C_{t,\mathrm{Y}}$	#				#				#			
$(1+r)^{-1}C_{t+1,\mathrm{Y}}$	#	#			#	#			#	#		
$-D_{t,\mathrm{Y}}$												

FIGURE 4: Production Coefficients.

	H_X	S_X	H_Y	S_Y
$C_{t,X}$	+	+	−	−
$(1 + r)^{-1}C_{t+1,X}$	+	+	−	−
$-D_{t,X}$	+	+	−	−
$C_{t,Y}$	−	−	+	+
$(1 + r)^{-1}C_{t+1,Y}$	−	−	+	+
$-D_{t,Y}$	−	−	+	+

FIGURE 5: Destruction Coefficients.

(iii) *Matrix* **C**′ *Destruction Coefficients* (See Figure 5). Impounding in ceteris paribus the vector of exogenous events and terrain,[16] this matrix cross-classifies as coefficients the six arguments of **A** shown as rows, each divided by the military activities of each country shown as columns. The sign pattern shown for the coefficients indicates whether activities shown in the headings either defend or destroy the elements of **A**.

The linearization introduced for expository purposes implies certain simplifications which are not essential to the argument.[17] The outcome of war on each of the components of **A** is the simple sum of each type of military activity weighted by the coefficients of **C**. There is no intrinsic distinction here between offensive and defensive weapons; defense or offense consists in the allocation of each of the military instruments shown as column headings. Offensive capabilities are negative numbers indicating a reduction in the argument vector **A**, and defensive capabilities are positive.

(iv) *Matrix* **D**: *Military Activities Coefficients* (See Figure 6). This matrix cross-classifies coefficients of the 4 military activities by the 24 allocations that are required to produce them. Within each block the H and S rows and the K, L, N columns with H and S subscripts will correspond to the allocations of the factors of production to military use by each country. Indicating positive entries by # and leaving zero entries blank this will be a block diagonal matrix. Military allocations to the civilian sector will be taken to be zero, setting aside spillover effects and recent Soviet attempts to apply military technology to civilian production (Cooper, 1987).

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[16] These influences could be introduced explicitly as a set of additional matrix "filters."

[17] The linear form of the destruction process could be replaced with a more general functional form for which the coefficients of **C** become partial derivatives. It should be noted that description of war by *any* well-defined function implies that the outcomes depend on the military instruments alone, independent of the order in which they occur. The analog to extended transitivity in the theory of consumer demand suggests a consistency to warfare that might not always be satisfactory, but is necessary for the optimization procedure displayed in this paper.

Country X Resource Allocations

	1	2	3	4	5	6	7	8	9	10	11	12
	K_C	K_I	K_H	K_S	L_C	L_I	L_H	L_S	N_C	N_I	N_H	N_S
H_X			#				#				#	
S_X				#				#				#
H_Y												
S_Y												

Country Y Resource Allocations

	13	14	15	16	17	18	19	20	21	22	23	24
	K_C	K_I	K_H	K_S	L_C	L_I	L_H	L_S	N_C	N_I	N_H	N_S
H_X												
S_X												
H_Y			#				#				#	
S_Y				#				#				#

FIGURE 6: Military-activities coefficients.

(v) *Vector F: Allocation Vector.* The vector of allocations of the three factors to four uses by two countries is a 24-element column vector which appears as the column headings of matrices **B** and **D**.

Now we may write the utility function in terms of **F** as

$$\mathbf{U} = (\mathbf{U}_X, \mathbf{U}_Y)' = (\mathbf{MU}_X, \mathbf{MU}_Y)\mathbf{A} = (\mathbf{MU}_X, \mathbf{MU}_Y)(\mathbf{B} - \mathbf{CD})\mathbf{F} \tag{18}$$

(vi) *Constraints.* The system of constraints on country $i\,(i = \mathrm{X, Y})$ is given to us in principle by the identities given previously in (6), but now we must relate them to the allocation vector **F**.

(a) The capital stock allocated to its four uses in year $T\,(T = t, t + 1)$ is equal to the stock in the previous period, minus the depreciation, plus the capital formation and, minus war destruction.

$$\begin{aligned} K_{iT} &= K_{iCT} + K_{iIT} + K_{iHT} + K_{iST} \\ &= K_{iT-1} - dK_{iT} + k(K_{IT}, L_{IT}, N_{IT}) - z(H_{iT}, S_{iT}) \end{aligned} \tag{19}$$

(b) Natural resources stock is similar

$$N_{iT} = N_{iCT} + N_{iIT} + N_{iST} = N_{i,T-1} - qN_{iT} - w(H_{iT}, S_{iT}) \tag{20}$$

(c) Labor allocated in T is

$$L_{iT} = L_{iCT} + L_{iHT} + L_{iHT} + L_{iST} = p(P_{i,T-1} - g(H_{iT}, S_{iT}) + QC_{iT} \tag{21}$$

For simplicity, we map these expressions as functions of **F** in the following linear fashion. Let $\mathbf{R}_T$ be the 6-by-8 block-diagonal matrix of resource use whose diagonal blocks are

$$\mathbf{R}_{iT} = \begin{pmatrix} K_{iCT} & K_{iIT} & K_{iHT} & K_{iST} \\ L_{iCT} & L_{iIT} & L_{iHT} & L_{iST} \\ N_{iCT} & N_{iIT} & N_{iHT} & N_{iST} \end{pmatrix} \tag{22}$$

and let $\mathbf{s}^*$ be the block diagonal matrix of depreciation and depletion of resources whose blocks are $\mathbf{s}_i^*$

$$\text{(23)} \qquad \mathbf{s}_i^* = \begin{pmatrix} 1+d & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1+q \end{pmatrix}$$

The product $\mathbf{s}^*\mathbf{R}_T$ is the matrix of adjusted factor uses and, when post-multiplied by the summation vector of ones, must be equal to the stock carried over from the previous period, adjusted for further production and destruction.

Grouped by country, define the preexisting stock of factors for each nation as the six element column vector $\mathbf{V} = (K_{i,T-1}, pP_{i,T-1}, N_{i,T-1})'$. To this we add the increment due to investment, consumption and, for completeness, the null increase in natural resources. The block-diagonal matrix $\mathbf{S}$ has for each nation the structure $\mathbf{s}_i$.

$$\text{(24)} \qquad \mathbf{S}_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \left[\text{the product } S_i \begin{pmatrix} I \\ C \\ N \end{pmatrix}_i\right]$$

yields the gross increase in factor supply. We express the latter vector in terms of the allocation vector $\mathbf{F}$, by the intermediate block diagonal matrix $\mathbf{M}$ whose elements are $\mathbf{M}_i$. $\mathbf{M}_i$ is a matrix of coefficients of production of (I, C, N) per unit allocation of the elements of $\mathbf{F}$. Thus, the blocks of $\mathbf{M}$ are

$$\text{(25)} \qquad \mathbf{M}_i = \begin{pmatrix} m_{Ii} \cdots\cdots\cdots m_{I12} \\ m_{Ci} \cdots\cdots\cdots m_{C12} \\ m_{Ni} \cdots\cdots\cdots m_{N12} \end{pmatrix} \qquad \left[\text{where } \mathbf{M} = \begin{pmatrix} \mathbf{M}_X & 0 \\ 0 & \mathbf{M}_Y \end{pmatrix}\right]$$

The first row of each block consists of the capital production coefficients. The second row is identical with the first row of matrix $\mathbf{B}$, minus their elements of the second row of the product $\mathbf{CD}$ to account for the destruction of consumer goods. The last row is identically zero.

To account for the destruction of resources we consider the (6-by-4) matrix $\mathbf{N}$ of destruction of capital, the labor force and natural resources by military activities. As in the case of destruction of elements of the utility function, this matrix will contain positive and negative elements depending on whether the effect is defensive or offensive. The second row of this matrix will draw elements from the third row of $\mathbf{CD}$ to account for the effect of casualties on the labor force. These losses have to be multiplied by p to adjust the loss in population for reduction in the active participating labor force. In turn, the military activities must be converted into resource allocations by the (4-by-24) matrix $\mathbf{D}$ defined earlier.

With these tedious accounting procedures we can now define the constraint system in matrix terms as

$$\mathbf{R}_T = \mathbf{s}^{*-1}[\mathbf{V} + (\mathbf{SM} - \mathbf{ND})\mathbf{F}] \tag{26}$$

9. OPTIMIZATION

Now we are in a position to formulate the problem in compact terms. Each nation is to choose its twelve components of the allocation **F** so as to maximize its own utility function subject to the system of constraints. Even though it might be the case that a rational nation does not gain satisfaction from the distress of its opponent as such, the utility functions of the nations are interdependent; the allocation of each nation represents a possibility of reducing the utility of the other by force or by requiring it to allocate resources other than what it would do if not threatened. Hence, whether or not that force is employed in any period, the optimal choice of the second nation is to reallocate resources from consumption in the present and future to increase military elements in **DCF**.

Since these military activities can be used in an offensive as well as defensive fashion, the attempts to purchase security by one nation threatens the other and constitutes a negative external effect to its safety. The **DC** matrix product shows how military activities enter as negative externalities. The negative externality necessitates an attempt at security through further military allocation. The course of the resulting arms race may be foreseen and calculated by the participants; or, in the absence of a treaty to reduce the external costs through arms control, it may proceed in a self-reinforcing fashion as the unintended result of individual national attempts to protect its population.

Generalizing the interactions of all the utility-maximizing decisions in this light we can state the problem of both countries as the maximization program

$$\underset{\mathbf{F}}{\mathrm{Max}}\ (U_{\mathrm{X}}, U_{\mathrm{Y}}) \tag{27}$$

such that

$$\mathbf{R}_{\mathrm{F}} = \mathbf{s}^{*-1}[\mathbf{V} + (\mathbf{SM} - \mathbf{ND})\mathbf{F}]$$

The point is that even though the choice of the portion of **F** is made by each country in terms of its own utility function in light of the resources it has available, those decisions are interconnected with the optimization decisions of the other. That is to say, military power cannot strictly be defined in the technical military sense of the term through the choice of instruments H and S. Rather, these instruments are intermediate goods, to be evaluated through their effect on utility in the two nation interactive process of optimization under conditions of external cost.

Military power—and by extension programs and negotiations to avoid its use—cannot be decided apart from the choices made in the rest of the social structure. A military-power function, such as we described in Part A of this paper, is therefore defined only locally in a region close to equilibrium of the nonmilitary elements.

The simplified diagrams—and proposals for MEL—in Part A are based on positing military power as a technically defined function (MPF), maximized subject to the dedication of resources to armaments as a military production-possibility frontier (MPPF). These are now seen to plausible under special conditions.

(i) The possibility of producing military instruments in various combinations (MPPF) are given and reflective of the resource endowments of each country. A *reasonably* sufficient condition for this to be the case is given by the approximation that the military expenditures are large enough to dominate the factor endowments of each country. At the same time, the continuation of deterrence in the form of achievement of a common isoquant of MPF, requires that it is possible to shift resources between the civilian and military sectors to ensure that the MPPF curves of opposing countries interest.

(ii) Locally the military-instrument mix is mapped continuously into the utility function of each country so that the level curves of the MPF can be defined as a locus in which the utility level of both nations are unchanged as a result of a change in military-instrument choice. As we noted in our earlier papers, if there are convexities, singularities, or catastrophes in this mapping then the MPF fails to be weakly concave and any sort of rational optimization procedure is threatened.

(iii) The dynamics of interaction between the parties—the lag structure of response—is not so unstable as to lead the participants so far from equilibrium that both deterrence and optimization become meaningless. If that were the case, then rational optimization is replaced by behavioral dynamic systems modeled by structural differential equations and ad hoc parameters.

(iv) As a practical matter, there exists a sufficient degree of complementarity in the alternative "bundles" weapons systems, so that it is possible to consider them in sufficiently aggregated way to conduct a rational choice. For example, the present dialog between the U.S. and the U.S.S.R. over the linkage of nuclear versus conventional weapons implies such a grouping of weapons akin to our *H* and *S* distinction. It should be noted that this problem is much reduced under the MEL proposal since it is not the weapon aggregations that need to be negotiated but the total expenditures.

However convenient and suggestive the formulation of military power, in general it must be merged with the total national optimization process. The distinction between war and peace is artificial and disappears altogether insofar as our model of optimization specifies an interior maximum. Every allocation, even current consumption, has military implications insofar as the resources it claims might otherwise have gone to the military. Current consumption motivates labor-force participation and productivity through Q and thus increases the level of output and the capacity to allocate goods to military power in the future on the part of the originating country. This positive and negative military effect is more pronounced in decisions to accumulate capital. What distinguishes economic warfare from other programs of economic activity is the specific linkage of weapons and other military activities acquisition; in light of the C and D matrices country X makes its choice of military instruments directly in response to those of Y in order to limit their effectiveness and hence maximize its own utility. In the final analysis, this direct route of acquisition of military instruments in the present period is not the only one which counts. The historic Soviet preoccupation with capital accumulation at the expense of consumption as a long-run means to military power illustrates this point.

Whatever the particular technique, given the objective utility function and resource limitations, standard methods of constrained maximization reveal (1) the system of implicit prices of the elements of the utility function; and, (2) the dual shadow prices of the factors of production in the neighborhood of the optima chosen by each nation.

From (1) it is possible to determine the trade-off which each nation is willing to make among present consumption, future consumption and death to some of its citizens. It is unlikely that the choice will be a corner in which $D = 0$.[18] That does not deny the possibility of other corners. But recent experience with, say, terrorism and counter-terrorism in "limited wars," "police actions" and so forth makes it seem likely that the optima are interior solutions of one or another sort and that both economic and military war are almost always with us.

From (2) it is possible to construct a cost function in terms of those shadow prices and to address the question of how the one nation may achieve a given level of its own utility and the utility of the other nation with minimum cost. In the case, we have examined that the initial stock of resources is fixed and the choices are made only by political figures. The leaders maximize their utility and the cost is an ex post statistic as the result of the shadow price dual of their preferences. If, however, they are not the only decision makers, but must match their shadow price to the (labor) market or other signals of their populations willingness to work, save or die, then the equilibrium is more complicated and more nearly akin to that of a utility-maximizing firm.

The decisions of the leadership constitute a derived demand curve for the factors of production at alternative "prices," so that "equilibrium" occurs where these prices match those of the population. The reason that we have placed "equilibrium" in quotation marks is that the optimum allocation for each country defines its own apparent maximum utility and that of the other nation. The fact that these choices are not made by each nation independently of the choices of the other is the central point to which we must now return.

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[18] The unwillingness of society to choose options leading to zero deaths is evidenced by the public preference for traffic fatalities rather that reduced automobile travel.

10. EQUILIBRIUM NEGOTIATIONS

We first consider the situation which we will call equilibrium in which both parties choose those allocations in year T which are "expected" by the other in their own optimization decisions. Remembering that the 24-element vector **F** consists of the 12 allocations of each nation, this amounts to saying that the optimal **F** vector is the same for both countries.

The term equilibrium is chosen faute de mieux because we do not mean the absence of conflict (even military conflict); nor do we imply a position of rest, since the allocations in t and $t + 1$ will be different and may continue to vary along some time path in the future. Certainly the position is not Pareto optimal, in light of the negative externality entailed in the search for security. Rather, equilibrium means that the parties are not mistaken as to the actions of the other.

(i) *Negotiations are possible* as long as interdependent "hate" does not appear in either utility function; marginal utilities are nonnegative for all the arguments of both utility functions; and, when traced through the string of transformations detailed in our accounting exercise, the marginal utilities of each country with respect to the military activities of the other are nonpositive and at least one is strictly negative for each nation. (This last would exclude any symbiosis of the "military-industrial" establishments gaining in payoff as a result of the level of their putative enemy, and influencing policy in their own interests.)

(ii) An *irrational state of war* exists when negotiations are possible and not carried out. A state of irrational *economic war* exists when the D argument in the utility function is zero for both parties. An irrational *military war* exists when $D > 0$. That is to say irrational war exists when $D > 0$ and negotiations are possible and not carried out.

Negotiations will be carried out provided both parties continue to increase their utility by reductions in military activities (and quite possibly "economic" reallocation and even transfers) provided that: (a) the cost of information is not prohibitive, there must not be "Catch 22" situations involving inspection and bargaining over military activities which are presumed to be secret until negotiations are successful and those activities are eliminated; and (b) the trajectory of successive negotiations follows a path of successive equilibria in the sense defined previously. The situation we are describing is similar to a repeated game of prisoners' dilemma in which the parties learn of their partners' reactions and are thus able to communicate and make de facto binding agreements.

11. DISEQUILIBRIUM

Somewhat anachronistically, a great deal more has already been written about disequilibrium Cournot duopoly types of international conflict than about equilibrium states. This is because the issue under disequilibrium becomes conjectural or cognitive which tends to be modeled in behavioral Richardson ways. Each nation has a propensity to react to the perceived activities or rates of change of activities of its antagonist thus generating dynamic feed back processes over time.

The important questions then are how each nation will react to the allocations of other; under disequilibrium circumstances it cannot simply optimize its own utility with respect to the present allocation of the other since the other nation is changing its course with respect to the first in such a way as to continue to leave their allocations inconsistent. Then if negotiations are to be successful they must include a return to equilibrium so that the process described previously can be invoked. This may or may not be possible depending on the dynamics of the reaction of each nation to the

Formally, at equilibrium optimum **F** allocations, each nation (say X) takes the resulting military activities of the other as consistent with its own. Then unambiguously *both* would gain from a differential reduction in its opponent's military activities, (S_Y, H_Y), because for X (and mutatis mutandis for Y)

$$dU_X = \frac{\partial U_X}{\partial S_Y} dS_Y + \frac{\partial U_X}{\partial H_Y} dH_Y \qquad \text{and} \qquad \left(\frac{\partial U_X}{\partial S_Y}, \frac{\partial U_X}{\partial H_Y}\right) < 0$$

Both would gain by mutual disarmament if sign (dS_X, dH_X) = sign (dS_Y, dH_Y). If, however, the allocations of each country are not in equilibrium, then X might expect that its allocations leading to (S_X, H_X) might well influence (S_Y, H_Y), then there are reaction functions such that

$$dU_X = \left[\frac{\partial U_X}{\partial S_Y}\left(\frac{\partial S_Y}{\partial S_X}\right) - \frac{\partial U_X}{\partial H_Y}\left(\frac{\partial H_Y}{\partial S_X}\right)\right] dS_X + \left[\frac{\partial U_X}{\partial S_Y}\left(\frac{\partial S_Y}{\partial H_X}\right) + \frac{\partial U_X}{\partial H_Y}\left(\frac{\partial H_Y}{\partial H_X}\right)\right] dH_X$$

The quantities in parentheses are *expectational* in nature and in any case may be of either sign and any magnitude. These "conjectural variations" may or may not converge to equilibrium. Specifically, country X might react to a military buildup in Y by reducing its own military in the hope of "appeasing" the other side; it might increase its own so as to maintain "parity"; or it might envision unacceptable conditions in the future in light of its economic inability to maintain parity and carry out a rapid build-up and "first strike" while it has a positive probability of a successful military outcome.

The Richardson differential-equation models involved fixed propensities to react to the actions of others in order to explain the arms races leading to World War I. Some years ago, one of the authors offered a dynamic model of the Cold War based on different perceptions of reality as influenced by the ideological belief systems of the U.S. and the U.S.S.R. (Wolfson, 1968, 1973). The claim to usefulness of all the behavioral models of this genre rests on the inability of the parties to optimize, either because of rigidity in the performance or perception, or because optimization fails due to disequilibrium in the sense we have defined it.

12. INTERDEPENDENT UTILITIES

A detailed discussion of this alternative is beyond the scope of the present paper. Suffice it to say that if the perceived utility of Y enters as an argument into that of X then matters become more complicated and even more dangerous. Continuing our formalism, if the utilities of countries are interdependent then the previous expressions contain the additional terms

$$\frac{\partial U_X}{\partial U_Y}\frac{\partial U_Y}{\partial S_X}dS_X + \frac{\partial U_X}{\partial U_Y}\frac{\partial U_Y}{\partial H_X}dH_X$$

These are positive for $(dS_X, dH_Y) > 0$, if, in addition to the previous specifications, hate enters into utility

$$\frac{\partial U_X}{\partial U_Y} < 0$$

Consequently the sign of dU_X and dU_Y resulting from increases in the acquisition of military instruments cannot be determined a priori. Hostile interdependence of utility offers incentives for a punitive arms race employing military and economic weapons. War is then neither more nor less irrational hatred itself.

13. CONCLUSIONS: THE MILITARY-POWER FUNCTION REVISITED

When is the MPF model used in Part A strictly correct? MPF is first of all a mapping from (H, S) from one's own country, and as such it is an intermediate good from allocation vector **F**. Thus the ultimate correct analysis of what a country does, includes all of the **F** allocations, including the interaction with the other country.

What is it mapping to? The answer is the utility of one's own country, assuming that nonmilitary allocations are held constant and that there is a condition of equilibrium as defined above between the ultimate allocations **F** of the two countries. Then the allocations that directly and indirectly lead to military power—including the reactions of the opposing country which in equilibrium are known and considered "netted out"—show how much the satisfaction of the "own" country is increased by those allocations.

MPF is then a partial entity, strictly defined under conditions of equilibrium as specified, and *has dimensions of the utility of the own country*. Therefore each country may be thought of as attempting to maximize military power only within the forgoing caveats and partial structure. Clearly then the frontier MPPF is also a partial entity which impounds those portions of **F** that lead to (H, S) while leaving the others constant for the moment. A change in the things held constant for both MPF and MPPF then can be treated in these partial representations as a parametric shift in the other allocations of resources of both countries or a shift in tastes and preferences (utility functions of the ruling decision makers and their constituents) and technical progress in military and civilian production technology.

The two functions are not strictly defined for *conditions out of equilibrium*. We are dealing with dynamic (Richardson style) reaction processes which are inconsistent with the definition of utility employed here since they depend on perceptions and represent conditions in which the maximization of such utility is irrelevant. So the analysis that we present, even though it is a two-period and, therefore, dynamic model in a limited sense, acquires strict meaning close to equilibrium where our concept of utility applies.

That does not mean that our simple model of disarmament is useless. On the contrary, we have defined the domain over which it is strictly useful, and as a pragmatic matter we have a notion as to how far away from equilibrium this partial two-period dynamics can stray and still be relevant. For instance, it would seem to be useful now, but probably would not be if large-scale military war broke out, if there were dramatic military technology breakthroughs, or if a country were undergoing dramatic political or economic changes. Nevertheless, these sorts of limitations are common to most economic models. We conclude that our rough and ready pragmatism of Part A remains a useful point of departure.

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